J. Fergusson's Library 16 Nov. 1802

# DARY's MISCELLANIES:

*Being, for the most part,*

## A Brief COLLECTION of *MATHEMATICAL THEOREMS,*

From divers Authors upon these Subjects following.

*I. Of the Inscription and Circumscription of a Circle.*
*II. Of plain Triangles.*
*III. Of spherical Triangles.*
*IV. Of the projection of the Sphere* in plano.
*V. Of Planometry and the Centre of Gravity.*
*VI. Of Solid Geometry. And* (therein) Gauging.
*VII. Of the Scale of Ponderosity,* alias, *the Stilliard.*
*VIII. Of the four Compendiums for quadratique Equations.*
*IX. Of Recreative Problems.*

By *Michael Dary.*

See Montucla's Hist. de Math. T. III p. 3

*London,* Printed by *W. G.* and sold by *Moses Pitt* at the *White-Hart* in *Little-Britain*, *Tho. Rookes* at the *Lamb* and *Ink-bottle* in *Gresham-Colledge*, and *Wil. Birch* at the *Bible* in *New-Cheapside* in *Moor-fields.* 1669.

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# TO THE READER.

*Courteous Reader*,

THOu haſt here preſented to thy view and cenſure a few Mathematical Notes, moſt whereof have lain by me many years; and the reaſon of their ruſhing into the Publick in this homely dreſs, is, for that I and ſome others have been traduced and derided in a Book lately publiſhed, Entituled, *A Guide to the Young Gager*, put forth by ſeveral Authors: In the tail of which Book there is a whole Broad-ſide (intricate, prepoſterous, inartificial, and moſt prodigiouſly erroneous) diſgorged by the *Lieutenant* or *Bringer-up* againſt a Book, Entituled, *The Art of Practical Gauging*; which Book

I am ſorry to ſee ſo incumbred with Preſs-Faults, though they were none of mine: But moſt of theſe Guns we ſhall Charge again, and turn them upon this our *Lieutenant*, for we ſcorn to give an Anſwer to his invective Examination, till we have firſt examined him.

In his *Stereo.prop.* pag.6.pr.2.he tells how to find the Solidity of a Fruſtum Pyramide whoſe baſes are parallel but not alike: The word *Fruſtum Pyramide* I cannot underſtand, but if he had ſaid Fruſtum of a Pyramide, he would have been underſtood by every judicious man: But the Solid there ſpoken of (as I am informed) might very fitly have been called a *Priſmoid*, for it is a kind of a Priſm; for although the Sides thereof ſhould be continued, they would never be included or terminated in one point, as the Pyramide is; therefore, why Fruſtum Pyramide?

For the conſtruction of this his Fruſtum Pyramide, he bringeth (as an induction of particulars) four Pyramides, four Priſms, and a Parallelepipedon, that is in all five Priſms; (for a Parallelepipedon is a Priſm) but in reality there is but two Priſms, and the

the four Pyramides, which are indeed but one Pyramide, in the largeſt ſenſe it is capable of, both baſes being parallel but not alike, each being a rectangle.

In his Propoſition pag. 105. (which he proſecutes in pag. 106, 107, 108.) the ſtreſs of his Argument is weak and infirm (I paſs by the Preſs-Fault pag. 105. $z+\cdot z$ for it ſhould be $z-2$) for pag. 106. he ſaith $z=\frac{1}{5}$ that is $z$ is $\frac{1}{5}$ and A,$\frac{1}{5}$; theſe numbers make good the Queſtion: But ſtay, though we ſhould grant $z=\frac{1}{5}$, it is yet to demonſtrate, that $z$ is $\frac{1}{5}$ and A,$\frac{1}{5}$; which I am not bound to tell him how to do: But it is evident the man made haſt to the 108 page, to fling dirt in the face of *Van Schooten* (for he doth not care who he doth beſpatter) and it is manifeſt, if you compare this laſt page with his Title page, which ſaith, *Particularly intended for Gaging*; let any man judge, whether this Propoſition have any relation at all to Gaging. But we would have him to know, that we can perform this Propoſition and two other, of good uſe for the rational Ordinates of the Circle and Hyperbola, without caſting dirt in the face of any perſon.

Prop. 1. $xx+yy=zz$. quef. $x$? $y$? $z$?
Sol. $x=aa-bb$. $y=2ab$. $z=aa+bb$.

Prop. 2. $xy+yy=zz$. quef. $x$? $y$? $z$?
Sol. $x=2ab+bb$. $y=aa$. $z=aa+ab$.

Prop. 3. $xx+xy+yy=zz$. quef. $x$? $y$? $z$?
Sol. $x=aa-bb$. $y=2ab+bb$. $z=aa+ab+bb$.

In all thefe three Solutions you may put $a$ and $b=$ any two numbers taken at pleafure.

Moreover in Page 107. his Note for Progreffions is invalid, and of no force: For

| | | |
|---|---|---|
| the Sum | of the Sum and Difference of any two numbers is equal to the | greateft. |
| the Diff. | double of the ——— | leaft. |

From whence the Argument is clear by affumption, for he affumes the fame greateft number twice: Therefore the Sum of the Sum and Difference muft be the fame that it was before; to wit, the double of that fame greateft number: So it is evident that

that there is no need of Unity for the firſt Term of this Progreſſion as he intimates: If he ſhall ſay his Note is true notwithſtanding my declamation, then let him ſhew a reaſon why he doth amuſe his Reader with ſuch a foolery. Are theſe the Men of a ſound judgement which they ſpeak of in their Preface? The like general Theorem may be laid down in Geometrical Progreſſions, but no need of Unity for the firſt Term.

The Fact { of the Fact & quote } greateſt.
{ of any two numbers }
{ is equal to the }
The quote { Square of the —— } leaſt.

But this I ſhall not inſiſt upon.

In his invective Examination, Pag 1. he ſaith, That our Equation is five times greater than it needs: If he could have ſaid it had required five times more work in the operation, he had ſaid ſomething; but to ſee how full he is fraught with Envy, for this very Propoſition hath been commended by divers Artiſts in this City, for the contrivance of it, becauſe it doth accommodate both the Spheroid and the Parabolical Spindle with one and the ſame

Diviſor, and the Operation is very near as ſhort as the old way for the Spheroid.

From the firſt page to the twelfth he is wholely taken up with inveighing againſt the Table of Segments; in which I ſee there are many Preſs Faults, but theſe Faults (as I ſaid before) are none of my Faults: For the making of the Table I gave this Rule, $6,28318853 : 0,017453 A - S : \times Q (= K$, which I ſhall demonſtrate.

It is manifeſt from *Archimedes*, *Snellius*, and other Authors, that if the Radius of a Circle be Unity, that then the Area of that Circle is 3,14159266 too much, or 3,14159265 too little: Alſo from the ſame Authors it holds, that the Area of a Sector is equal to the Fact or Rectangle of the Radius into $\frac{1}{2}$ the Arch or Baſe of the Sector: Then if you put A = to the Arch, S = to the Sine of that Arch, it muſt hold $1 \times \frac{1}{2}A$ = the Area of the Sector, and $1 \times \frac{1}{2}S$ = the Area of the Triangle in the Sector; but the Sector leſs the Triangle is equal to the Segment of the Circle in that Sector: So then $\frac{1}{2}A - \frac{1}{2}S$ = any Segment of a Circle whoſe Radius is Unity. And

And by the 11. and 12. Prop. *Partis Cyclicæ* of *Leotaud's Examen* of the Quadratures of *Greg.* of St. *Vinc.* it holds, As the Area of the Circle given 3,14159265, is to $\frac{1}{2}$ A — $\frac{1}{2}$ S; ſo is Q, the Quadrature or Area of any Circle propoſed, to K, its like Segment. Then multiplying both the firſt and ſecond terms by 2, in Jimbals (for ſo he calls the Symbols in deriſion) it will ſtand thus;

6,283185 3) : A—S ::× Q :( = K.

Now becauſe A ſtands in Degrees and Centeſmes, it muſt be reduced to the ſame parts that S ſtands in, which may be done thus; As 360 deg. the whole Peripheria in the Parts of A, is to 6,283185 3 the whole Peripheria in the parts of S; ſo is A, to 0,017453A: therefore 0,017453A, comes in the room of A, and the Rule ſtands thus;

6,283185 3):0,017453A—S:: ×Q(=K which was to be demonſtrated.

In page 12. this inſulting Scoffer ſaith, *This may ſerve for inſtruction to Segment-Makers for the future, to inform them whether their Work goes on regularly or not.*

Reader

Reader, I hope thou haſt more underſtanding than to take this *Bringer-up* for one that modeſtly makes publick pure *Geometry* (as he ſpeaks in his Preface.) But if thou doſt, I ſhall undeceive thee: For it is apparently true, that if a Table of the Segments of a Circle ſhall be differenced never ſo often, they have no equal Differences; but he ſeems to intimate that they have: For if he mean otherwiſe, I would fain know of him the Habitude of that rank of Differences by which he will prove the truth of a Table of Segments. But methinks I hear the Reader object and ſay, That if a Table of Segments be differenced far enough, the Differences at laſt will be equal: Stay there, herein lies the deception; for thoſe Figures that appear to be equal, are only frontier-Figures, and if you make the Table of Segments a good company of places larger, you will then ſee the ragged Regiment that ſtands behind theſe Frontiers. This he ought to have told his Reader, otherwiſe he publiſheth (not pure, but) very impure *Geometry*.

In page 13. he proceeds to the Trial of the

The Tables of Wine, and Beer and Ale; and the Table for dividing the Gaging Rod, and he concludes them alſo badly calculated, as may be proved (ſaith he) by taking the ſecond or third Differences.

*What? the ſecond, or the third?* 'Tis a marvel when his hand was in, he had not put in the *fourth* Differences too: *O ye Blind Guides!* know ye not that the Tables for Wine, Ale and Beer, are capable but only of the firſt and ſecond Differences; which I prove thus for the Wine: the conſtruction of the Wine Table is the Square of any Diameter in Inches, divided by 883. Then if you put D = a Diameter propoſed (do you ſee now) in Jimbals it will ſtand thus:

| | | |
|---|---|---|
| DD+0D+ 0 | 1 Diff. | 2 Diff. |
| DD+2D+ 1 | 2D+1 | |
| DD+4D+ 4 | 2D+3 | 2 |
| DD+6D+ 9 | 2D+5 | 2 |
| DD+8D+16 | 2D+7 | 2 |

All to be divided by 883. And if you ſhould interpole never ſo often, you ſhall find no more but the ſecond Differences.

But

But it seems they will have the second and third Differences come in, for *'tis all one to* Anthony *who Kisseth* Dorothy.

But what if our *Lieutenant* should say he did intend the third Differences for the Trial of the Gaging-Rod Table: But certainly he did not, (out of doubt he would not be so unkind to his *Young Gager*, to leave him thus in the dark *guideless*,) if he did he is caught in an evil Net, for there he might as well have said the thirteenth as the third, for that Table is the ordinates of a Parabola standing at equal distances. This may serve for an instruction to our Table-Tryers, how they burn their Fingers again: Let them learn how to be Table-Makers before they turn Table-Tryers.

*Courteous Reader*, I am sorry I have held thee in this discourse so long: Now let me address my self to thee, that we may understand one another. In Chap. 6. Prop. 1. thou wilt meet with a Solid, which I call a *Prism*; by which word is there meant a Solid having two Bases, equal, parallel, alike, and alike situate, and

and in the Peripetaſma a Right Line may be every where applied, from one Baſe to another.

A *Pyramide* of the ſame Baſe and height with the Priſm, is $\frac{1}{3}$ thereof: And in the Peripetaſma a Right Line may be every where applied, from the Baſe to the Vertex.

A *Pyramidoid* of the ſame baſe and height with the Priſm, is ſome certain portion thereof; as if it be parabolical, it is $\frac{1}{2}$; if it be ſpherical, it is $\frac{2}{3}$: And in the Peripetaſma a Right Line may be no where applied from the Baſe to the Vertex.

For my Diviſion, it is ſuch as is uſed by others: As for Example, 24): 37 + 41 : × 4 (= 13. and may be read thus; 24 dividing 37 more 41, multiplyed by 4, equal to (or quotes) 13: Or it may be read by analogie thus: As 24, is to 37 more 41; ſo is 4, to 13. If thou meeteſt with ſome Diviſions that ſtand double lined, they were things that had lain by me a good while, and I would not ſtand to alter them.

So

So craving thy favourable construction, where any thing hath slipt amiss, for it was not the intent of him who desires, if he were able, to be

Thine to serve thee,

*Michael Dary.*

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Errata

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# ERRATA.

THe five laſt Pages are wrong numbered, 42, 43, 44, 45, 46 they ſhould be 44, 45, 46, 47, 48; and then pag. 24. lin. 20. for of its read of all its pag. 26. lin. 15. for A=S read A—S p. 29. l. 5. for L.= read =L. p. 31. l. 5. for 0,159 read 0,16 p. 33. l. 1. for If read 8. If p. 36. l. 12. for Conjugate read rectangular Conjugate l. 16. for rectanguled read rectangular l. 18. for rectanguled read rectangular p. 37. l. 8. for =D read =D, p. 43. l. 3. & 4. for ſides read lines p. 45. l. 14. for let be read let it be, and for terms in read terms: In p. 47. l. 16. for number read any number. Pag. 36. for Convex Begirter or Zone you may read Peripetaſma.

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The

# The Contents.

CHap. 1. *Of the Inſcription and Circumſcription of a Circle.* pag. 1.
Chap. 2. *Of Plain Triangles.* pag. 10.
Chap. 3. *Of Spherical Triangles.* pag. 13.
Chap. 4. *Of the projection of the Sphere in* plano. pag. 20.
Chap. 5. *Of Planometry and the Centre of Gravity.* pag. 23.
Ch. 6. *Of ſolid Geometry.* p. 29.
Chap. 7. *Of the Scale of Ponderoſity,* alias *the Stilliard.* p. 43
Chap. 8. *Of the 4 Compendiums for quadratique Equations.* pag. 45.
Chap. 9. *Of recreative Problems.* pag. 47.

# Dary's *Miſcellanies.*

## CHAP. I.

### *Of the Inſcription and Circumſcription of a Circle.*

1. FOraſmuch as the *Ratio* of an Arch line to a right line is yet unknown, it is abſolutely neceſſary, that right lines be applyed to a Circle for the Calculation of Triangles wherein Arch lines come in Competition.

2. Right lines applyed to a Circle are

Chords, Sines, Tangents, Secants, and verſed Sines.

3. The Chord of an Arch is a right line extended from one end of that Arch to the other end thereof: The Sine is a right line drawn from one end of that Arch, Perpendicularly upon the Diameter drawn from the other end of that Arch: The Tangent is a right line touching one end of that Arch extended till it Concur with the Secant: The Secant is a right line extended from the Center of the Circle till it Concur with the Tangent: The Verſed Sine is a right line being a Segment of the Diameter, drawn from one end of that Arch till it be cut by a Perpendicular (i. e. the Sine) from the other end of that Arch.

4. It is to be noted by this Definition in Prop. 3. that the Chord of an Arch is common to two Arches, one of them being the Complement of the other to a whole Circle; and likewiſe the Verſed Sine is common to two Arches, one of them being the Complement of the other to a whole Circle: But the Sine of an Arch is common to two Arches, one of them

them being the Complement of the other to a Semi-circle.

5. As the Sum of two Sines is to their difference, so is the Tangent of the $\frac{1}{2}$ Sum of those Arches, to the Tangent of their $\frac{1}{2}$ difference.

6. As the Sum of two Tangents, is to their Difference, so is the Sine of the Sum of those Arches, to the Sine of their Difference.

7. As the Sine of the Sum of two Arches, is to the Sum of their Sines, so is the difference of those Sines, to the Sine of their Difference.

8. If you put R = The Radius of a Circle, A = an Arch proposed, C = the Chord of that Arch, S = the Sine of that Arch, T = the Tangent of that Arch, and Z = the Secant of that Arch.

Then

$$\frac{S\ R}{Co.\ S} = T$$

$$\frac{R\ R}{Co.\ T} = T$$

$$\frac{RR}{Co,S} = Z$$

$$Z + T = T \text{ of } 45^{\text{deg.}} + \tfrac{1}{2}A$$
$$Z - T = T \text{ of } 45 - \tfrac{1}{2}A$$

$$2Z = \text{Sum} \begin{cases} T \text{ of } 45^{\text{deg.}} + \tfrac{1}{2}A \\ \text{and} \\ T \text{ of } 45 - \tfrac{1}{2}A \end{cases}$$

$$T = \text{Dif.} \begin{cases} T \text{ of } 45^{\text{deg.}} + \tfrac{1}{2}A \\ \text{and} \\ T \text{ of } 45 - \tfrac{1}{2}A \end{cases}$$

$$\frac{RR + RT}{R - T} = T \text{ of } 45^{\text{deg.}} + A$$

$$\frac{RR - RT}{R + T} = T \text{ of } 45^{\text{deg.}} - A$$

$$\frac{2RRT}{RR - TT} = T \text{ of } 2A$$

$$\sqrt{\frac{RRRR}{TT} + RR} - \frac{RR}{T} = T \text{ of } \tfrac{1}{2}A$$

9. If

9. If twice three Arches equi-different be propoſed, Then, as the Sine of one of the means, is to the ſum of the Sines of its Extreams; ſo is the Sine of the other mean, to the ſum of the Sines of its Extreams.

10. And hence, if a rank of Arches be equi-different, As the Sine of any Arch in that rank, is to the ſum of the Sines of any two Arches equally remote from it on each ſide; So is the Sine of any other Arch in the ſaid rank, to the ſum of the Sines of two Arches next to it on each ſide, having the ſame common diſtance.

11. Three Arches equi-different being propoſed, If you put Z = the Sine of the greater extream, Y = the Sine of the leſſer extream, M = the Sine of the Mean, m = the Co-ſine thereof; D = the Sine of the common difference, d = the Co-ſine thereof, and R = the Radius.

1. Then $Z + Y = \frac{2Md}{R}$

3. Then $Z - Y = \frac{2\,m\,D}{R}$

3. Then $Z\,Y = M\,M - D\,D$

4. Then $\frac{Z}{Y} = \frac{M\,d + m\,D}{M\,d - m\,D}$

12. From the last before going, it is evident, that if two thirds (i.e either the former, or the latter 60 deg. or the former 30 deg. and the latter 30 deg.) of the Quadrant be compleated with Sines: the remaining third part of the Quadrant may be compleated by Addition or Subduction onely.

13. If in a Circle, two right lines be inscribed cutting each other, The Rectangles of the Segments of each line are equal. And the Angle at the point of Intersection is measured by the Half-sum of its intercepted Arches.

14. If to a Circle two right lines be adscribed from a point without, The Rectangles of each line from the point assigned to the Convex and Concave are equall. And the Angle at the assigned point is measured by the half difference of its intercepted Arches

15. If in a Circle (or an Elipsis) three right lines shall be inscribed, one of them

them cutting the other two: Then the Rectangles of the Segments of each line ſo cut, are directed proportional to the Rectangles of the reſpective Segments of of the Cutter.

16. If a plain Triangle be inſcribed in a Circle, the Angles are one half of what their oppoſite ſides do ſubtend.

17. Therefore the Angles of a plain Triangle are equal to a Semi-circle.

18. And hence, if a Rectangled Triangle be inſcribed in a Circle, the Hypothenuſe thereof is the Diameter of the Circle.

19. As the Diameter of a Circle is to the Chord of an Arch; ſo is that Chord, to the verſed Sine of that Arch.

20. And hence, if from the right angle of a rectangled Triangle, a Perpendicular be let fall upon the Hypothenuſe, the Hypothenuſe is thereby cut according to the *Ratio* of the ſquares of the ſides.

21. If in a Circle, any plain Triangle be inſcribed, and a Perpendicular be let fall upon one of the ſides, from the oppoſite angular point; Then as that Perpendicular is to one of the adjacent ſides, ſo is the

other adjacent ſide, to the Diameter of the Circumſcribring Circle.

22. If a Circle be inſcribed within a plain Triangle, Then, as the Perimeter is to the Perpendicular; ſo is the Baſe on which it falleth, to the Radius of the inſcribed Circle.

23. If a Quadrilateral Figure be inſcribed in a Circle, and Interſect with Diagonals, The Rectangle of the Diagonals is equal to the two Rectangles of the oppoſite ſides.

24. If a Circle be both inſcribed and circumſcribed by two like ordinate Polligons; Then, as the Co-verſed Sine of the ſide of the inſcribed is to the Diameter, ſo is the Area of the Inſcribed to the Area of the Circumſcribed.

25. If an ordinate Polligon be both Inſcribed and Circumſcribed by two Circles; Then, as the Diameter of the Circumſcribed, is to the co-verſed Sine of the ſide of the Polligon; So is the Area of the Circumſcribed, to the Area of the Inſcribed.

26. In any right lined Figure, if a Circle be Inſcribed; Then, as the Periphe

ria

ria of the Circle, is to the Area thereof; So is the Perimeter of the right lined Figure, to the Area thereof. *Et Con.*

27. But in all Circles, as the Peripheria is to the Area, so is 2 to the Radius.

28. Therefore; In any right lined Figure, if a Circle be inscribed, as 2. is to the Radius; So is the Perimeter of the right lined Figure, to the Area thereof.

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CHAP.

## CHAP. II.

## *Of Plain Triangles.*

1. A Triangle is a Figure Comprehended of three ſides; and is either Plain or Spherical.

2. A Plain Triangle is that which is deſcribed on a Plain Surface, whoſe three ſides are right lines; and it is either right Angled or oblique Angled; and the oblique, is either obtuſe or acute.

3. If a line drawn from the top or vertex of a Triangle equally Biſecting the Baſe, be equal to the Biſegment, the Vertical Angle is a right Angle; if leſſer Obtuſe, if greater Acute.

4. In a Plain Triangle, a right line equally

equally Biſecting the Vertical Angle, cuts the Baſe directly according to the *Ratio* of the adjacent legs.

5. Any one ſide of a Triangle is leſs than the ſum, and greater than the difference of the other two ſides.

6. Any one ſide being continued, the exterior Angle is equall to the two interior Angles oppoſite.

7. In any right angled Plain Triangle, the ſum of the ſquares of the ſides containing the right Angle is equal to the ſquare of the Hypothenuſe.

8. In a Plain Rectangled Triangle any one of the ſides may be put for Radius, and the other ſides ſhall be Sines, Tangents, or Secant.

9. In any plain Triangle the ſides are directly proportional to the Sines of their oppoſite Angles, *Et Con.*

10. In any Plain Triangle, as the ſum of any two ſides is to their difference, ſo is the Tangent of the Half-ſum of their oppoſite Angles, to the Tangent of their Half-difference.

11. In any Plain Triangle, as the Baſe is to the ſum of the Legs, ſo is the

the difference of the legs, to the difference of the Segments of the Base, cut by a Perpendicular from the vertical Angle.

12. In any Plain Triangle, as the Base is to the sum of the legs; so is the Sine of $\frac{1}{2}$ the vertical Angle, to the Sine of the sum of $\frac{1}{2}$ the vertical Angle, and either of the Angles conterminate at the Base

13. In any Plain Triangle, as the Diameter is to the versed Sine of the vertical Angle;

So is the square of the sum of the legs } less by the square of the difference of the legs
To the square of the Base }

14. In any Plain Triangle, the fact of the legs and the Sine of their Angle, is equal to the fact of the Base, Petpendicular, and Radius.

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CHAP.

## CHAP. III.

## *Of Spherical Triangles.*

1. A Spherical Triangle, is that which is deſcribed on the ſurface of a Sphere.

2. The ſides of a Spherical Triangle, are Arches of three great Circles mutually interſecting each other.

3. The meaſures of Spherical Angles, are arches of great Circles, deſcribed from the Angular Pointsas their Poles, and ſubtending their Angles.

4. Thoſe are ſaid to be great Circles, which biſect the Sphere.

5. Thoſe Circles which cut each other at right Angles, the one of them paſſeth by the Poles of the other. *Et Con.*

6. The Diſtance of the Poles of two great Circles, is equal to the Angle comprehended by them.

7. The

7. The 3 Angles of any Spherical Triangle being given, there are likewiſe three ſides of another Spherical Triangle given, whoſe Angles are equall to the ſides of the former Triangle.

8. The ſum of the Sides of a Spherical Triangle are leſs than two Semi-circles.

9. The ſum of the 3 Angles of a Spherical Triangle, are greater than two right Angles, but leſs than Six.

10. Two Angles of any Spherical Triangle, are greater than the difference between the 3 Angle and a Semi-circle.

11. Any ſide being continued, the Exterior Angle is leſs than the two Interior oppoſite ones.

12. In any Spherical Triangle, the difference of the ſum of two Angles and a whole Circle, is greater than the difference of the third Angle and a Semi-circle.

13. A Spherical Triangle, is either Rect-angular, or Oblique-angular.

14. A Rect-angular Spherical Triangle, is that which hath one right Angle at the leaſt.

15. The Leggs of a Rect-angular Sphe-

Spherical Triangle, are of the ſame affection with their oppoſite Angles.

16. In a Rect-angular Spherical Triangle, if either legg be a Quadrant, the Hypothenuſe is alſo a Quadrant; but if both be of the ſame affection, the Hypothenuſe ſhall be leſs than a Quadrant; if of different affections, then greater: *Et Con.*

17. In a Rect-angular Spherical Triangle, if either of the Angles at the Hypothenuſe be a right Angle, the Hypothenuſe ſhall be a Quadrant; but if both of the ſame affection, it ſhall be leſs, if different then greater: *Et Con.*

18. In a Rect-angular Spherical Triangle, either of the Oblique-angles is greater than the Complement of the other, but leſs then the difference of the ſame Complement to a Semi-circle.

19. An Oblique angular Spherical Triangle, is either Acute-angular, or Obtuſe-angular.

20. An Acute angular Spherical Triangle, hath all its Angles Acute.

21. An Obtuſe angular Spherical Triangle, hath all its Angles Obtuſe or

mixt,

mixt, *viz.* Acute and Obtuſe.

22. In an Acute angular Spherical Triangle, each ſide is leſs than a Quadrant.

23. In an Oblique angular Spherical Triangle, if two Acute Angles be equal, the ſides oppoſite to them ſhall be leſs than Quadrants; if Obtuſe, greater.

24. In an Oblique angular Spherical Triangle, if two Acute Angles be unequal, the ſide oppoſite to the leſſer of them ſhall be leſs than a Quadrant; but if Obtuſe, the ſide oppoſite to the greater, ſhall be greater.

25. In every Oblique angular Spherical Triangle, if the Angles at the Baſe be of the ſame affection, the Perpendicular drawn from the top of the Vertical Angle ſhall fall within the Triangle; if different, without.

*In*

## *In Oblique angular Spherical Triangles, if a Perpendicular be drawn from the Vertical Angle, to the oppoſite ſide, (continued if need be.)*

*Conſectary* 1. The Co-ſines of the Segments of the Baſe are directly proportional to the Co-ſines of the ſides of the Vertical Angle: *Et Con.*

*Con.* 2. The Co-ſines of the Angles at the Baſe are directly proportional to the Sines of the Vertical Angles: *Et Con.*

*Con.* 3. The Sines of the Segments of the Baſe are reciprocally proportional to the Tangents of the Angles Conterminate at the Baſe: *Et Con.*

*Con.* 4. The Co-ſines of the Vertical Angles are reciprocally proportional to the Tangents of their ſides: *Et Con.*

## *Axioms for the Solution of Spherical Triangles.*

### *Axiom* 1.

IN Rect-angular Spherical Triangles having the ſame Acute angle at the Baſe: The Sines of the Hypothenuſes are proportional to the Sines of their Perpendiculars.

### *Axiom* 2.

In Rect-angular Spherical Triangles having the ſame Acute angle at the Baſe: The Sines of the Baſes and the Tangents of the Perpendiculars are proportional.

### *Axiom* 3.

In all Spherical Triangles, the Sines of the Angles are directly proportional to the Sines of their oppoſite ſides: *Et Cor.*

*Axioms*

*Axiom* 4.

In all Spherical Triangles, as the fact of the ſides containing the Vertical angle, is to the ſquare of the Radius; So is the fact of the Sines of the $\frac{1}{2}$ ſum, and the $\frac{1}{2}$ difference of the Baſe and difference of the Legs, to the ſquare of the Sine of $\frac{1}{2}$ the Vertical Angle.

*Or thus,*

In all Spherical Triangles, having added all the 3 ſides together, find the difference betwixt each ſide and their half ſum: And then,

As the fact of the Sines, of the $\frac{1}{2}$ ſum of all the ſides, and the difference of the ſide oppoſite to the Vertical Angle, is to the fact of the Sines of the differences of the containing ſides from the ſaid $\frac{1}{2}$ ſum, ſo is the ſquare of the Radius to the ſquare of the Tangent of $\frac{1}{2}$ the Vertical Angle.

## CHAP. IV.

# *Of the Projection of the Sphere in Plano.*

### 1. *Orthographically.*

IF a Sphere be by a Plain cut into two Hemiſpheres, and the Eye be placed at an infinite diſtance, Vertically to one of the Hemiſpheres; then a right line infinitly extended from the Eye, to any aſſigned point in the Spherical ſurface of that Hemiſphere, ſhall porject the aſſigned point upon the Plain: And the diſtance upon the Plain, from the apex of the Hemiſphere to the projected point, is equal to the ſine of the Arch from the Vertex of the Hemiſphere to the aſſigned point; the Radius of the Sphere being put for Radius.

2. *Sterio-*

## 2. *Steriographically.*

If a ſphere be by a plain touch't, and the eye be placed in the ſpherical-ſurface Diametrically oppoſite to the touch-point; Then a right line infinitly extended from the eye to any aſſigned point in the Spherical ſurface ſhall project the aſſigned point upon the Plain: And the diſtance upon the Plain from the touch-point to the projected point is equal to the Tangent of $\frac{1}{2}$ the Arch from the touch-point to the aſſigned point: The Diameter of the Sphere being put for Radius.

## 3. *Gnomonically.*

If a Sphere be by a Plain touch'd, and the Eye be placed at the Center of the Sphere; Then a right line infinitly extended from the Eye to any aſſigned point, in the Spherical ſurface (whoſe diſtance from the touch-point is leſs than a Quadrant) ſhall project the aſſigned point upon the Plain: And the diſtance upon the plain from the touch-point to the pro-

jected point is equal to the Tangent of the Arch from the touch-point to the assigned point: The Radius of the Sphere being put for Radius.

## *To project a Meridian line upon any Horizontal plain.*

1. Having prepared a piece of Metal or Wood, and made it a true Plain, and in some convenient point thereof (taken as a Centre) erected a Gnomon of sufficient length, at right angles to the Plain: This done, fix the Plain truly Horizontal

2. If you take the Suns Co-altitude (i. e. his distance from the Zenith) three times in one day, and (according to the Steriographical projection (having a line of Tangents by you) set off from the Centre of your plain or foot of the Gnomon, the Tangent of $\frac{1}{2}$ each Arch upon his respective Azimuth, or shadow (continued if need be) made by the Gnomon at that instant when the Co-altitude is taken, you will insert three points upon the Plain.

3. If

3. If you find out the Centre to those three inserted points, then a rigth line infinitely extended by this Centre found, and the Centre of the Plain, or foot of the Gnomon, is the true Meridian line: Which was to be projected.

## CHAP. V.

## *Of Planometry, and the Center of Gravity.*

1. IN any plain Triangle, the fact of the Base, and the Perpendicular, is equal to the double of the Area of that Triangle.

2. In any Plain Triangle, as the Diameter is to the Sine of any one of the angles, so is the fact of the adjacent Legs to the Area.

3. And hence, in how many soever Plain Triangles, having one angle equal

in Common, the facts of their ſides including the common Angle, are directly proportional to their Areas. *Et Con.*

4. In any plain Triangle, as the fact of the Diameter and the Sine of any one Angle, is to the ſquare of the oppoſite ſide; ſo is the fact of the Sines of the other two Angles to the Area.

5. If you put P = the Semi-perimeter of any plain Triangle, and D, $\mathcal{D}$, d, = the reſpective differences accruing by Subduction of each perticular ſide from the Semi-perimeter, and the Area = A, Then: 1) $\sqrt{D\,\mathcal{D}\,d\,P}$ (= A.

6. A Triangulate (i. e. any right lined figure) is Compoſed of Triangles, and the Triangles are leſs by two than the number ſides, and the Diagonals are leſs by three: And the Area thereof is equal to the Area of its Triangles.

7. If you put 1 = the Radius of a Circle, then the Area (and alſo the Semi-peripheria) ſhall be = 3,1416 *fere*: according to *Van Culen*, *Snellius*, and *Hugenius*.

8. If you put D = the Diameter of a Circle, P = the Peripheria, and A = the Area, then:

1,27324) D D (= A

12,5637) P P (= A

1) 3,1416 D (= P

1) √ : 12,5637 A : (= P

3,1416) P (= D

1) √ : 1,27324 A : (= D

9. If you put K = the Area of the Segment, or Kant of a Circle; V = the Verſed Sine in the Segment, D = the Diameter of the Circle, or Q equal to its Area; alſo if you put D) 2 V (= u, a Verſed Sine (to be found in a Table of Verſed Sines, the Diameter being 2,000 *&c.*) whoſe reſpective Arch in Degrees and Decimals being doubled, you may call A, and the correſpondent Sine of A you may call S, Then:

8) : 0,017453 A — S : × D D (= K

Or

6,2831853) : 0,017453 A — S : × Q (= K

10.

10. Again, if you put K or k = the greater or leſſer Segment of a Circle, cut by a Chord-line, D = the Diameter of the Circle y = the difference of the Segments of the Diameter cut at right Angles by the foreſaid Chord-line, alſo if you put D) y ( = S, a Sine (to be found in the Common Table of Sines, the Radius being 1,0000 &c.:) whoſe reſpective Arch in Degrees and Decimals being doubled you may call A, and the correſpondent Sine of A you may call S, Then:

8): 3, 1416 + 0,017453 A + S: × DD ( = K.

8): 3, 1416 — 0,017453 A = S: × DD ( = k.

Another way:

1, 27324 ): 0, 5 + 0, 00277 A + 0, 16 S: × DD ( = K.

1, 27324 ): 0, 5 — 0, 00277 A — 0, 16 S: × DD ( = k.

11. If you put Z = the Area of a Zone of a Circle intercepted between the Dia-

Diameter and Chord-line parallel to it, D = the Diameter of the Circle, B = the breadth of the Zone, Then:

10D—15 B): 10D—16B: × BD( > Z.

10D—16 B): 10D—17B: × BD( < Z.

12. If you put k = the Area of the Segment of a Circle (not greater then a Semi-circle,) C = the distance of the Centre of Gravity from the apex of the Segment, r = the Radius, c = the Chord of the Segment, and u = the Versed Sine in the Segment, Then:

9 r—6 u): 6 r—4 u: × c u ( < k.

9 r—6 u): 6 r—5 u: × c u ( > k.

25 r—15 u): 10 r—6 u: × u ( < G.

25 r—15 u): 10 r—5 u: × u ( > G.

13. If you put H = the Area of an Hyperbola, G = the distance of the Centre of Gravity from the apex of the Hyperbola, a = the Axis, B = the Base, r = the

the Semi-tranſverſe Diameter (between the Vertex of the Hyperbola, and the Center of the aſſymtoptes) Then:

9 r + 6 a) : 6 r + 4 a : × B a ( > H.

9 r + 6 a) : 6 r + 3 a : × B a ( < H.

25 r + 15 a) : 10 r + 6 a : × a ( > G.

25 r + 15 a) : 10 r + 5 a : × a ( < G.

14. If you put D = the diſtance of the Center of Gravity of a Plain right lined Triangle from one of the Angular points, and l = the right line from that Angular point Biſecting the oppoſite ſide, Then: 3) 2 l (= D.

15. If you put D = the diſtance of the Center of Gravity of the Sector from the Center of the Sector, c = the Chord, r = the Radius Biſecting A = the Arch, Then: 3 A) 2 r c (= D.

16. If you put L = the diſtance of the Center of Gravity of a, ( = the leſſer of two Superficial figures propoſed) and l = the diſtance of the Center of Gravity of A, ( = the greater of two Superficial Figures

Figures propoſed) from the common Center of Gravity of both the foreſaid Figures, and D = the whole diſtance of their reſpective Centers of Gravity, Then:
A + a ) D A (L. = and A + a) D a (= l.

17. As an unite is to the Radius, ſo is the exceſs of the 3 angles above a Semicircle (in a Spherical Triangle) to the boſſed ſurface of that Triangle. The exceſs is to be taken in the ſame parts, as is the Radius.

18. If a Sphere be encloſed in a Cylinder, and that Cylinder be cut with plains paralel to its baſe; then the Intercepted rings of the Cylinder are equal to the Intercepted ſurfaces of the reſpective Segments of the Sphere.

## CHAP. VI.

## *Of Solid Geometry.*

1. If you put Pr = A Priſm, B = one of its Baſes, and P = the Perpendicular

pendicular height of the Prisme, Then:

1) B P ( = Pr.

2. If you put Fr = the Frustum of a Piramide or a Cone intercepted between two Plains Paralel cutting the axis, B = the greater Base, b = the lesser Base, and P = the Perpendicular height of the Frustum Then,

3) : B + ✓ B b + b : × P ( = Fr.

3. If you put Fr = the Frustum of a Spherical Piramidoid, Sphere, or Spheroid, Intercepted between two Plains parallels, one of them passing by the Centre, B = the greater Base, b = the lesser Base and P = the Perpendicular height of the Frustum, Then:

3) : 2 B + b : × P ( = Fr.

4. If you put Fr = the Frustum of a Porabolical Piramidoid or Conoid Intercepted between two Plains parallel, cuting the axis, B = the greater Base, b = the lesser Base, and P = the Perpendicular height of the Frustum: Then

2) : B + b : × P ( = Fr.

5. If you put S = a Sollid made by Rotation, R = the Radius or neareſt diſtance, between the Centre of Gravity of

A

A = the begetting Figure, and a right line in the same Plain assigned (without) for an axis, and p = the Peripheria of a Circle whose Radius is unity, Then:
1) A R p, (= S. or Thus 0,159) AR) = S.

6. If you put Fr = the Frustum of a Parabolical Spindle, intercepted between two plains Parallel, one of them passing by the Centre, B = the greater Base, b = the lesser Base, P = the Perpendicular height of the Frustum, Then:
15) : 8 B + 7 b : × P ( = Fr.

7. In the 2, 3, and 4 Propositions: (which Propositions are general for Piramids or Cones, Piramidoids or Conoids, of what Base soever) If it will serve your turn to find onely the Cone, Conoids, and Parabolical spindle, when their Bases are Circles, it may be delivered thus,

1. If you put Fr = the Frustum of a Cone incepted between to Plains parallel, cutting the Axis at right Angles, D = the Diameter of the greater Base, d = the Diameter of the lesser Base, P = the Perpendicular hight of the Frustum, and make S = D + d, Then:
3,82) : S S — D d : × P ( = Fr.

2. If

2. If you put Fr = the Fruſtum of a Sphere or Spheroid, intercepted between two Plains paralel, one of them paſſing by the Center Cutting the Axis at right Angles, D = the Diameter of the greater Baſe, d = the Diameter of the leſſer Baſe, and P = the Perpendicular height of the Fruſtum, Then :

3,82) : 2 D D + d d : × P ( = Fr.

3. If you put Fr = the Fruſtum of a Parabolical Conoid, intercepted between two Plains paralel cutting the Axis at right Angles, D = the Diameter of the greater Baſe, d = the Diameter of the leſſer Baſe, and P = the Perpendicular height of the Fruſtum, Then :

2,54) : D D + d d : × P ( = Fr.

4. If you put Fr = the Fruſtum of a Parabolical Spindle, intercepted between two Plains Paralel, one of them paſſing by the Center cutting the Axis at right Angles, D = the Diameter of the greater Baſe, d = the Diameter of the leſſer Baſe, and P = the Perpendicular height of the Fruſtum, Then :

1 9,1) : 8 DD + 7 dd : × P ( = Fr.

8. If

If you put Fr = the Fruſtum of a Sphere intercepted between two Plains parallel, one touching and the other cutting the Sphere, d = the Diameter of the Baſe, and P = the perpendicular height of the Fruſtum, Then:

2,54648) : dd + 1 ⅓ PP : × P (= Fr.

This Rule will alſo hold if it were the fruſtum of a Spheroid, putting d d = the fact of the right angled Conjugates in the baſe.

9. If you put Fr = the fruſtum of a Cone intercepted between two Plains parallel, one of them being fixed the other moveable, D = the Diameter of the fixed baſe, p = the perpendicular height of the fruſtum, and d = the increment or decrement of any two Diameters at one Inch diſtance in the perpendicular, Then:

3,82) : 3DD ± 3Ddp + ddpp : × p (= Fr.

10. A Coopers common Cask, that is ſuch as are round at their heads (and not eliptical as ſome Oyl Cask are) being

Propoſed, if you put D = the Diameter at the bouldge, d = the Diameter at the heads, G = the diagonal from the middle of the bongue-hole to the bottom of either of the heads, L = the length of the Veſſel, and make S = D + d, Then:

$L = \surd : 4GG - SS:$

$G = \frac{1}{2}\surd : LL + SS:$

$S = \surd : 4GG - LL:$

$D = S - d.$

$d = S - D.$

11. According to the Equated Circle, now in uſe, if you put Q = the Quantity of Liquor in a Coopers common Cask being filled totally or partly, the axis being poſited parallel to the horizon, the Veſſel being taken as the middle Fruſtum of a parabolical Spindle intercepted between two Plains parallel, equidiſtant from the Centre cutting the axis at right angles, D = the Diameter of the bouldge,

V =

V = that proportion of this Diameter which is wet, d = the Diameter at the heads, and P = the perpendicular height or length of the Veſſel, alſo you ſhall put D ) V ( = N, which N abuts you to K = a Segment to be found in the Table of the Segments of a Circle whoſe Area is unity: Or if you have not by you a Table of Segments you may find K by *Chap.* 5. *Prop.* 9. and then if you divide by 19,1 you ſhall have cubical Inches, if by 4412, Wine Gallons; if by 5386, Ale or Beer Gallons; and the Rule will ſtand thus:

19,1 ) : 8 D D + 7 d d : × P × K ( = Q.

But if the ſaid Veſſel be taken as the fruſtum of a Spheroid intercepted, *&c.* Then inſtead of : 8 D D + 7 d d : you ſhall put : 10 D D + 5 d d : And yet the foreſaid Diviſors hold true to all intents and purpoſes.

12. Concerning the *Cylindroid*, *&c.* in its ſeveral kinds and ſeveral fruſtums.

By the word Cylindroid (in this place) is meant a Sollid contained under three

Surfaces *(i. e.)* two Plains parallel and a Convex begirter, whereof the two Plains parallel are called the Baſes, and are both Circles or both Ellipſis, or elſe one a Circle and the other an Ellipſis, and the Convex Surface is called the Zone; in which Zone there may be every where a right line applyed from any point in one baſe to ſome point in the other: and if ſuch a Cylindroid be cut with two Plains meeting in the Centers of both Baſes, cutting (or rather inſerting) conjugate Diameters in both Baſes, Then:

If you put C = the ſollid Content of a Cylindroid.
A & B aloft = the two rectangled conjugate Diameters.
G & H below = the two rectangled conjugate Diameters.
A & G oppoſite = the two correſpondent Diameters.
B & H oppoſite = the two correſpondent Diameters.
Alſo P = the perpendicular height of the Cylindroid, Then:

3,82):

$3,82) : \overline{A + \frac{1}{2}G : \times B + : G + \frac{1}{2}A : \times H} : \times P (= C.$

13. To cut the Cylindroid with divers Plains parallel, the Plains being of one common diſtance, and that diſtance being taken for Unity in the lineal Menſuration of the Cylindroid: To do this from that Baſe wherein G and H are Conjugates, you ſhall make P) A — G (= D and P) B — H (= d, and Then:

$:3G + 1\frac{1}{2}D : \times H + : 1\frac{1}{2}G + D : \times d =$ firſt Fr

$:3G + 4\frac{1}{2}D : \times H + : 4\frac{1}{2}G + 7D : \times d =$ firſt diff.

$3DH + : 3G + 12D : \times d =$ ſecond diff.

$6Dd =$ third diff.

All to be divided by 3,82.

If it ſhall ſo happen that A = B and G = H, it is the truſtum of a Cone, Then:

$3GG + : 3G + D : \times D =$ firſt Fr.

$3GG + : 9G + 7D : \times D =$ firſt diff.

: 6 G + 12 D : × D = second diff.
6 D D = third diff.
All to be divided by 3,82.

If it shall so happen that B = H, Then:
: 3 G + 1 ½ D : × H = first Fr.
: 3 G + 4 ½ D : × H = first diff.
3 D H = second diff.
All to be divided by 3,82.

If it shall so happen that B=H=G, Then:
: 3 G + 1 ½ D : × G = first Fr.
: 3 G + 4 ½ D : × G = first diff.
3 D G = second diff.
All to be divided by 3,82.

If it shall so happen that A = B = G = H, Then it is a Cylinder: to be divided by 3,82.

14. Now instead of P = the perpendicular of the whole height of the Solid, if you shall put p = the perpendicular height of any part thereof, from that Base wherein G and H are Conjugates, and C = the solid content of the Cylindroid at that height,

height, then it holds as in the margin :

But if ſuch a ſolid have not its Zone made by Circles or Ellipſies but by four flat ſides at right angles to the foreſaid Conjugates, then it is a *Priſmoid* : Nevertheleſs the Rules before preſcribed hold to all intents and purpoſes, if you take away the Diviſor 3,82 and in the room thereof place the Diviſor 3.

15. This laſt Propoſition to find the Content of the Cylindroid or Priſmoid at any height or depth may be alſo performed by the Table of figurate Numbers following, thus :

1. Having got the firſt Fruſtum, firſt, ſecond, and third differences (if there be ſo many) of your Solid, multiply them by the reſpective numbers in the Table at that height or depth.
2. Add all theſe Products together into one Sum, having reſpect to the ſigns + and — (if they ſhall be ſo ſigned) and this Total is the Content of your Solid at the height or depth propoſed.

*A*

3,82) : 3 GH $\pm$ : 1 $\frac{1}{2}$ HD + 1 $\frac{1}{2}$ Gd : × p + Ddpp : × p (= C.

*A Table of Figurate Numbers for the speedy Collecti-*
*third Differences, in finding the Content of the*

| Depth. | 1 Frac. | 1 Diff. | 2 Diff. | 3 Diff. |
|---|---|---|---|---|
| 01 | 1 | 00 | 000 | 0000 |
| 02 | 1 | 01 | 000 | 0000 |
| 03 | 1 | 02 | 001 | 0000 |
| 04 | 1 | 03 | 003 | 0001 |
| 05 | 1 | 04 | 006 | 0004 |
| 06 | 1 | 05 | 010 | 0010 |
| 07 | 1 | 06 | 015 | 0020 |
| 08 | 1 | 07 | 021 | 0035 |
| 09 | 1 | 08 | 028 | 0056 |
| 10 | 1 | 09 | 036 | 0084 |
| 11 | 1 | 10 | 045 | 0120 |
| 12 | 1 | 11 | 055 | 0165 |
| 13 | 1 | 12 | 066 | 0220 |
| 14 | 1 | 13 | 078 | 0286 |
| 15 | 1 | 14 | 091 | 0364 |
| 16 | 1 | 15 | 105 | 0455 |
| 17 | 1 | 16 | 120 | 0560 |
| 18 | 1 | 17 | 136 | 0680 |
| 19 | 1 | 18 | 153 | 0816 |
| 20 | 1 | 19 | 171 | 0969 |
| 21 | 1 | 20 | 190 | 1140 |
| 22 | 1 | 21 | 210 | 1330 |
| 23 | 1 | 22 | 231 | 1540 |
| 24 | 1 | 23 | 253 | 1771 |
| 25 | 1 | 24 | 276 | 2024 |

*on and exact Correction of the first, second and Cylindroid or Prismoid.*

| Depth. | 1 Fruſ. | 1 Diff. | 2 Diff. | 3 Diff. |
|---|---|---|---|---|
| 26 | 1 | 25 | 300 | 2500 |
| 27 | 1 | 26 | 325 | 2600 |
| 28 | 1 | 27 | 351 | 2925 |
| 29 | 1 | 28 | 378 | 3276 |
| 30 | 1 | 29 | 406 | 3654 |
| 31 | 1 | 30 | 435 | 4060 |
| 32 | 1 | 31 | 465 | 4495 |
| 33 | 1 | 32 | 496 | 4960 |
| 34 | 1 | 33 | 528 | 5456 |
| 35 | 1 | 34 | 561 | 5984 |
| 36 | 1 | 35 | 595 | 6545 |
| 37 | 1 | 36 | 630 | 7140 |
| 38 | 1 | 37 | 666 | 7770 |
| 39 | 1 | 38 | 703 | 8436 |
| 40 | 1 | 39 | 741 | 9139 |
| 41 | 1 | 40 | 780 | 9880 |
| 42 | 1 | 41 | 820 | 10660 |
| 43 | 1 | 42 | 861 | 11480 |
| 44 | 1 | 43 | 903 | 12341 |
| 45 | 1 | 44 | 946 | 13244 |
| 46 | 1 | 45 | 990 | 14190 |
| 47 | 1 | 46 | 1035 | 15180 |
| 48 | 1 | 47 | 1081 | 16215 |
| 49 | 1 | 48 | 1129 | |
| 50 | 1 | 49 | 1178 | |

3. The conſtruction of this Table is very eaſie, for you may ſee it is nothing but a Collection of Unites: But to make the reſpective numbers in this Table for the firſt, ſecond, and third Differences to any Depth propoſed, without any gradual Collection, this is the Rule.

If you put D = the Depth

Then 1) D — 1 (= 1 Diff.

2) D D — 3 D + 2 (= 2 Diff.

6) DDD — 6DD + 11 D — 6 (= 3 diff.

Or if you put D = the firſt Diff.

Then 1) D (= 1 Diff.

2) D D — D (= 2 Diff.

6) DDD — 3DD + 2 D (= 3 Diff.

16. The ſymetry of like Superficies and like Solids. Superficies are like, and Solids are like, if the Angles be equal in number and quantity, [illegible]

17. Like

17. Like Superficies are one to another as the Squares of their Correſpondent Sides: Like Solids are one to another as the Cubes of their Correſpondent Sides.

## CHAP. VII.

## *Of the Conſtruction and Uſe of the Scale of Ponderoſity (commonly called) the* Still-yard.

1. A Right Line reſting on a fulciment equiponderate being propoſed: Then if any ponderoſity ſhall be applyed to a point of Pendency in that Line; it ought to be underſtood, that that Ponderoſity is tranſplanted from the Fulciment to that point of Pendency: But if any Ponderoſity ſhall be withdrawn or taken away from a point of Pendency in that Line, it ought

ought to be underſtood that that Ponderoſity is tranſplanted from the point of Pendency to the Fulciment.

2. A Right Line reſting on a Fulciment equiponderate being propoſed: If divers Ponderoſities ſhall be pendantly applyed on ſundry points of that Line, ſo that the ſaid line be equiponderate again: Then the facts (of each Ponderoſity by its tranſplantation from the Fulciment) on this ſide the Fulciment, are equal to the facts on that ſide the Fulciment.

3. A Right Line, reſting on a Fulciment, equiponderate by divers Ponderoſities, pendant in ſundry points of that line, being propoſed: If two of the Ponderoſities pendant ſhall be tranſplanted, ſo that the ſaid line be equiponderate again: Then the facts of each Ponderoſity by his diſtance run in tranſplantation are equal.

4. If a Stillyard or Scale of Ponderoſity, or (as the *Dutch* call it) the *Roman* Beam, be true, (*i. e.* doth give the truth) in two points (the farther diſtant the better) it is true in all the points of pendancy throughout, the Diviſions being equal.

CHAP.

## CHAP. VIII.

## *Of the four Compendiums for quadratique Equations.*

IN theſe four Compendiums you have both the affirmative and negative Roots ſymbolically expreſt, $z$ being the unknown Symbol in each Equation, (but made known by each Solution) $a$ being the known factor in the firſt term; $b$ being the known factor in the ſecond term; $r$ being the reſolvend.

But if at any time an Equation ſhall be propoſed, incumbred with vulgar fractions, let be reduced to its leaſt terms in whole numbers, if poſſible; if not, let it be reduced to its leaſt terms in Decimals; and then it will fall under one of theſe four Equations following.

In which Equations you muſt note, that if there be no known factor expreſt in the firſt term, then $a$ is underſtood to be unity: Furthermore, it is evident, that if any quantity

quantity ſhall be ſigned — that then t
the Square Root, or the Root of any ev
power of ſuch quantity ſo ſigned, is ine
plicable: Therefore when this ſhall ha
pen in the Solution of any Equation wha
ſoever, that Equation may be ſaid to b
impoſſible.

1. Equa. $+azz+bz=+r$

Sol. $2a) \pm\sqrt{:+bb+4ar:}-b\left(=\frac{+z}{-z}\right.$

2. Equa. $+azz-bz=+r$

Sol. $2a) \pm\sqrt{:+bb+4ar:}+b\left(=\frac{+z}{-z}\right.$

3. Equa. $+azz-bz=-r$

Sol. $2a) \pm\sqrt{:+bb-4ar:}+b\left(=\frac{+z}{+z}\right.$

4. Equa. $+azz+bz=-r$

Sol. $2a) \pm\sqrt{:+bb-4ar:}-b\left(=\frac{-z}{-z}\right.$

CHAP

## CHAP. IX. *Of Recreative Problems.*

Problem 1. *To find* a, b, c, *three numbers under this qualification.*

$aa = bb + cc$. quef. *a? b? c?*

Sol. $A = nn + 1$. $B = nn - 1$. $C = 2n$.

You may put $n =$ any number.

Probl. 2. *To find* a, b, c, *three numbers under this qualification.*

$ab + bb = cc$. quef. *a? b? c?*

Sol. $A = 2n + 1$. $B = nn$. $C = nn + n$.

You may put $n =$ any number.

Probl. 3. *To find* a, b, c, *three numbers under this qualification.*

$a + ab + bb = cc$. quef. *a? b? c?*

Sol. $A = 2n + 1$. $B = nn - 1$. $C = nn + n + 1$.

You may put $n =$ number.

Probl. 4. *To find* a, b, c, d, e, *five numbers under this qualification.*

$a + b = c$. $cc - a = dd$. $ee - b = ee$.

Quef. *a? b? c? d? e?*

Solution $A = \frac{n^4 + 4n^3 + 5nn + 2n}{: nn + 4n + 1 : (2)}$

$B = \frac{2n^3 + 5nn + 4n + 1}{: nn + 4n + 1 : (2)}$

C=

$$C = \frac{nn + 2n + 1}{nn + 4n + 1}$$

$$D = \frac{nn + n}{nn + 4n + 1}$$

$$E = \frac{n + 1}{nn + 4n + 1}$$

You may put $n =$ any whole number. But note, that every Equation is to be reduced to its least terms, if need require.

Probl. 5. *To find the Content of any Solid made by Rotation, if you can get the* ratio *of the Squares of its Ordinates, to any rank of Rectangles: This is the* Theorem.

If two ranks of Quantities or Numbers be propoſed (in any qualification or order whatſoever) having one common *ratio* between each pair of Correſpondents: Then in both ranks their Correſpondent Sums or Differences have the ſame common *ratio*. This Theorem holds as well in Superficies, if you can get the *ratio* of the Ordinates to any other rank of Quantities or Numbers.

*FINIS.*

# GAUGING EPITOMIZED:

OR,

*An Abbreviation of* Solid Geometry, *concerning the Business of* CASK-GAUGING, *taking a* Cask *in any of the Four Notions following. By* Michael Dary.

## *PREMONITION.*

IF you put D = the Diameter of the Bouldge, d = the Diameter of either of the Heads, and y = D — d; L = the length of the Axis of the Vessel, and C = the Content thereof, the dimension being taken in Inches.

*Prop.* 1. If a Cask be taken as the middle Frustum of a *Spheroid*, intercepted between two Planes parallel cutting the Axis at right angles: Then, 3,82) : 2DD + dd : ×L (= C.

*Prop.* 2. If a Cask be taken as the middle Frustum of a parabolick Spindle, intercepted between two Planes parallel, cutting the Axis at right angles: Then, 3,82) : 2DD + dd — 0,4yy : ×L (= C.

*Prop.* 3. If a Cask be taken as the middle Frustum of two parabolick Conoids abutting upon one Common Base, intercepted, *&c.* Then, 3,82) : DD + dd : ×1½L (= C.

*Prop.* 4. If a Cask be taken as the middle Frustum of two Cones abutting upon one Common Base, intercepted, *&c.* Then, 3,82) : DD + dd — ⅓yy : ×1½L (= C.

Now if you would have the Content in Beer Gallons, you must multiply 3,82 by 282, makes your divisor = 1077 *fere*: If you would have Wine Gallons, 3,82 × 231 makes your divisor = 883 *fere*. And from these two divisors, you may Calculate Tables for Wine or Beer Measure: For the square of any diameter in Inches divided by 883, is the Construction of the Wine Table; or by 1077, is the Construction of the Beer Table: And either of these Tables have their second differences equal, therefore they will be made by an easie Collection.

¶ *If a* Cask *be not full (the* Axis *being posited parallel to the* Horizon*) to find the quantity of Liquor contained in it.*

To do this, you ought to have a Table of the *Segments* of a Circle, whose *Area* is unity, the *Diameter* being divided into 10000 equal parts, and then this Approximation is the readiest hitherto used, which requireth this *Data*; the whole Content of the Cask, the *Diameter* at the Bouldge, and the wet Portion thereof; and the Proportion runs thus: As the whole *Diameter*, is to its wet Portion; so is the *Diameter* in the Table, (*i.e.* 10000,) to its like Portion: Which being sought in the Table of *Segments*, abutts you to a *Segment*, by which if you mnltiply the whole Content of the Cask, the Product is the Content of the remaining Liquor in the Cask.

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*Here followeth an Account of some* Addenda *to, and Faults escaped in,* Dary's Miscellanies.

P*Age* 4. *might have been added*, or T)RZ — RR(= T of ½A. *p.* 11. *l.* 10. *for* right anguled *r.* rectangular. *l.* 14. *for*

= 1077 *fere*: If you would have Wine Gallons, 3,82 × 231 makes your diviſor = 883 *fere*. And from theſe two diviſors, you may Calculate Tables for Wine or Beer Meaſure: For the ſquare of any diameter in Inches divided by 883, is the Conſtruction of the Wine Table; or by 1077, is the Conſtruction of the Beer Table: And either of theſe Tables have their ſecond differences equal, therefore they will be made by an eaſie Collection.

¶ *If a* Cask *be not full (the* Axis *being poſited parallel to the* Horizon*) to find the quantity of Liquor contained in it.*

To do this, you ought to have a Table of the *Segments* of a Circle, whoſe *Area* is unity, the *Diameter* being divided into 10000 equal parts, and then this Approximation is the readieſt hitherto uſed, which requireth this *Data*; the whole Content of the Cask, the *Diameter* at the Bouldge, and the wet Portion thereof; and the Proportion runs thus:

As the whole *Diameter*, is to its wet Portion; ſo is the *Diameter* in the Table, (*i.e.* 10000,) to its like Portion: Which being ſought in the Table of *Segments*, abutts you to a *Segment*, by which if you multiply the whole Content of the Cask, the Product is the Content of the remaining Liquor in the Cask.

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*Here followeth an Account of ſome* Addenda *to, and Faults eſcaped in,* Dary's Miſcellanies.

*Page* 4. *might have been added*, or TORZ—RR (=T of ½A. *p.* 11. *l.* 10. *for* right anguled *r.* rectangular. *l.* 14. *for* rectanguled *r.* rectangular. *p.* 14. *pr.* 7. *ſhould run thus*, The 3 Angles of any Spherical Triangle being given, there are likewiſe 3 ſides of another Spherical Triangle given, whoſe Angles are equal to the ſides of the firſt Triangle, if you take the Complement to 180 *deg.* of one of them, but moſt conveniently of the greateſt. *p.* 19. *l.* 1. *for* fact of the ſides *r.* fact of the ſines of the ſides. *p.* 20. *after the orthographical Projection ſhould have followed*, By this means all Circles of the Sphere perpendicular to the Plane, are projected into Right Lines, thoſe parallel to it into Circles of the ſame bigneſs, and all other into Ellipſes. *So after the Steriographical Projection*, By this means, all great Circles of the Sphere, paſſing thorough the Eye and Touch-point, are Projected into Right Lines, and all other into Circles, and the Angles in this Projection are equal to their Correſpondent Angles on the Sphere. *So after the Gnomonical Projection*, By this means, all great Circles on the Sphere perpendicular to the Plane, are Projected into Right Lines, all thoſe parallel to it into Circles, and all others into *Hyperbola's*, *Parabola's* or *Ellipſes*, accordingly as they cut or touch that Great Circle of the Sphere which is parallel to the Plane or do neither. *P.* 23. *l.* 15. *for* Legs *r.* Sides. *P.* 25. *l.* 2. & 4. *for* 12,56377 *r.* 12,5664. *P.* 27. *l.* 1. *for* and Chord *r.* and a Chord; *l.* 4. *for* —15B *r.* —16B. *P.* 28. *after Prop.* 15 *ſhould have followed*, The *Area* of a *Parabola*, is ⅔ of the circumſcribed *Parallelogram* of the ſame Baſe and Height; and the diſtance of its Centre of Gravity from its Vertex towards its Baſe, is ⅗ of its Altitude: Moreover in a *Semiparabola* the Centre of Gravity, is in a Right Line parallel to the Baſe, at ⅖ of the Height; and in a Right Line parallel to the Axis at ⅜ of the Baſe. *P.* 19. *Prop.* 18. *ſhould run thus*, If a Sphere be inſcribed in a Cylinder, and that Cylinder be Cut with Planes at Right Angles to its Axis, then the intercepted Surfaces of the Cylinder are equal to the intercepted Surfaces of the reſpective Segments of the Sphere. *P.* 31. *l.* 1. *for* the begetting *r.* the Area & the begetting; *l.* 13. *for* 8B+7b *r.* 8B+4√Bb+3b. *P.* 32. *l.* ult. *and P.* 35. *l.* 14. & 17. *for* 8DD+7dd *r.* 8DD+4Dd+3dd. *P.* 35. *Prop.* 12. *ſhould run thus*, By the word *Cylindroid* is meant a Solid contained under three Surfaces (*i.e.*) two Planes parallel and a Peripetaſma, whereof the two Planes parallel are the Baſes, and are both Circles or both Ellipſes, or elſe one a Circle and the other an Ellipſis, and a Right Line extended from the Centre of one Baſe to the Centre of the other, may be called the Axis of the Solid; and in the Peripetaſma a Right Line may be any where applied from Baſe to Baſe, being in the ſame Plane with the Axis of the Solid.

But if ſuch a Cylindroid be Cut with two Planes meeting in the Centers of both Baſes, Cutting (or rather Inſerting) rectangular Conjugate Diameters (or Axes) in both Baſes, then &c. *P.* 37. *l.* 2. *for* divers *r.* many. *P.* 39. *l.* 3. *for* Zone *r.* Peripetaſma; *l.* 6. *for* to the foreſaid Conjugates *r.* between themſelves, then it is a Priſm or Priſmoid. *P.* 42. *l.* ult. *for* quantity *r.* quantity, and their ſides proportional.

*This Table of Wine Meaſure ſhould have come in* Pag. 35. *of the* Miſcellanies.

are likewise 3 sides of another Spherical Triangle given, whose Angles are equal to the sides of the first Triangle, if you take the Complement to 180 *deg.* of one of them, but most conveniently of the greatest. *p.* 19. *l.* 1. *for* fact of the sides *r.* fact of the sines of the sides. *p.* 20. *after the orthographical Projection should have followed*, By this means all Circles of the Sphere perpendiular to the Plane, are projected into Right Lines, those parallel to it into Circles of the same bigness, and all other into Ellipses. *So after the Steriographical Projection*, By this means, all great Circles of the Sphere, passing through the Eye and Touch-point, are Projected into Right Lines, and all other into Circles, and the Angles in this Projection are equal to their Correspondent Angles on the Sphere. *So after the Gnomonical Projection*, By this means, all great Circles on the Sphere perpendicular to the Plane, are Projected into Right Lines, all those parallel to it into Circles, and all others into *Hyperbola's*, *Parabola's* or *Ellipses*, accordingly as they cut or touch that Great Circle of the Sphere which is parallel to the Plane or do neither. *P.* 23. *l.* 15. *for* Legs *r.* Sides. *P.* 25. *l.* 2. & 4. *for* 12,5637 *r.* 12,5664. *P.* 27. *l.* 1. *for* and Chord *r.* and a Chord; *l.* 4. *for* —15B *r.* —16B. *P.* 28. *after Prop.* 15 *should have followed*, The *Area* of a *Parabola*, is $\frac{2}{3}$ of the circumscribed *Parallelogram* of the same Base and Height; and the distance of its Centre of Gravity from its Vertex towards its Base, is $\frac{3}{5}$ of its Altitude: Moreover in a *Semiparabola* the Centre of Gravity, is in a Right Line parallel to the Base, at $\frac{3}{8}$ of the Height; and in a Right Line parallel to the Axis at $\frac{3}{8}$ of the Base. *P.* 19. *Prop.* 18. *should run thus*, If a Sphere be inscribed in a Cylinder, and that Cylinder be Cut with Planes at Right Angles to its Axis, then the intercepted Surfaces of the Cylinder are equal to the intercepted Surfaces of the respective Segments of the Sphere. *P.* 31. *l.* 1. *for* the begetting *r.* the Area of the begetting; *l.* 13. *for* 8B+7b *r.* 8B+4$\sqrt{}$Bb+3b. *P.* 32. *l.* ult. *and P.* 35. *l.* 14. & 17. *for* 8DD+7dd *r.* 8D+4Dd+3dd. *P.* 35. *Prop.* 12. *should run thus*, By the word *Cylindroid* is meant a Solid contained under thre Surfaces (*i. e.*) two Planes parallel and a Peripetasma, whereof the two Planes parallel are the Bases, and are both Circles or both Ellipses, or else one a Circle and the other an Ellipsis, and a Right Line extended from the Centre of one Base to the Centre of the other, may be called the Axis of the Solid; and in the Peripetasma a Right Line may be any where applied from Base to Base, being in the same Plane with the Axis of the Solid.

But if such a Cylindroid be Cut with two Planes meeting in the Centers of both Bases, Cutting (or rather Inserting) rectangular Conjugate Diameters (or Axes) in both Bases, then &c. *P.* 37. *l.* 2. *for* divers *r.* many. *P.* 39. *l.* 3. *for* Zone *r.* Peripetasma; *l.* 6. *for* to the foresaid Conjugates *r.* between themselves, then it is a Prism or Prismoid. *P.* 42. *l.* ult. *for* quantity *r.* quantity, and their sides proportional.

*This Table of Wine Measure should have come in* Pag. 35. *of the* Miscellanies.

| *The Names of Wine Vessels.* | Cubical Inches. | Pints. | Quarts. | Gallons. | Runlets. | Hogsheads. | Tercions. | Pipes or Butts. | Tuns. |
|---|---|---|---|---|---|---|---|---|---|
| 1 Tun | 58212 | 2916 | 1008 | 252 | 14 | 4 | 3 | 2 | 1 |
| 1 Pipe or Butt | 29106 | 1008 | 0504 | 126 | 07 | 2 | $1\frac{1}{2}$ | 1 | |
| 1 Tercion | 19404 | 0672 | 0336 | 084 | $04\frac{2}{3}$ | $1\frac{1}{3}$ | 1 | | |
| 1 Hogshead | 14553 | 0504 | 0252 | 063 | $03\frac{1}{2}$ | 1 | | | |
| 1 Runlet | 04158 | 0144 | 0072 | 018 | 01 | | | | |
| 1 Gallon | 00231 | 0008 | 0004 | 001 | | | | | |
| 1 Quart | 000$57\frac{3}{4}$ | 0002 | 0001 | | | | | | |
| 1 Pint | 000$28\frac{7}{8}$ | 0001 | | | | | | | |
| 1 Cubical Inch | 00001 | | | | | | | | |